

BIFURCATIONS OF A FORCED  
RAYLEIGH EQUATION

by

K.I. Taha

A thesis submitted for the degree of

Doctor of Philosophy

at the University of London

July, 1980

Department of Mathematics,  
Westfield College,  
University of London.

BOOK CHECKED

PHILADELPHIA UNIVERSITY LIBRARY  
ACQ. NO. 11577  
CLASS. NO. 510  
DATE 1980

## ABSTRACT

We discuss the bifurcations of a two-parameter averaged system of a forced oscillator of the form

$$\ddot{x} - \xi \dot{x} + \zeta x^2 \dot{x} + x + \alpha x^3 = \psi \cos \Omega t \quad ,$$

where  $\xi = 2$  ,  $\zeta = 8$  and  $3\alpha = 8\Omega$  .

Local bifurcations of Hopf and saddle-node type and global bifurcations of saddle connection type are found to exist and are investigated. The normal forms techniques of Takens are used to investigate the degeneracies of the system at certain exceptional parameter points. Furthermore, the limit cycle behavior of the system is discussed from the point of view of existence and uniqueness. Numerical calculations are also used to support the various theoretical results.

# CONTENTS

	Page
Chapter One	6
Introduction	6
1.1	6
The Basic Equation	6
1.2	11
Fundamental Definitions and Theorems	11
1.3	17
The Plan of the Thesis	17
Chapter Two	19
Bifurcation Set and Distribution of	19
Singularities	19
2.1	19
Introduction	19
2.2	21
Local Bifurcation Set $B_\ell$ : (Bifurcations	21
of Singular Points)	21
2.3	21
The Set $B_\ell^0$	21
2.4	24
The Set $B_\ell^1$	24
2.5	27
The Set $B_N$ (nodes, foci)	27
2.6	29
The Hopf Bifurcation on $B_\ell^1$	29
2.7	33
The Saddle-Node Bifurcation on $B_\ell^0$	33
2.8	41
Response Curves of the Vector Field	41
2.9	46
Distribution and Stability of Singular	46
Points in $(\omega, \rho)$ -Plane (Response Curves	46
Plane)	46
2.10	48
Distribution and Stability of Singular	48
Points in $(\omega, F)$ -Plane	48
2.11	53
The Global Bifurcation Set $B_G$	53
2.12	57
The Saddle-Node Bifurcation Set as a	57
Catastrophe Set	57

Chapter Three	Bifurcation of the Vector Field at the Exceptional Parameter Points	65
3.1	Introduction	65
3.2	The Vector Field at C	65
3.3	A Theorem of Takens' and its Application to the Bifurcation at C	69
3.4	The Vector Field at I	82
3.5	The Vector Field at D, O and G	87
Chapter Four	Limit Cycles	92
4.1	Introduction	92
4.2	The Existence of Periodic Solutions when $\rho < \frac{1}{2}$	92
4.3	The Uniqueness of Limit Cycles when $\rho < \frac{1}{3}$	97
4.4	The Non-existence of Limit Cycles when $\rho > \frac{1}{2}$	105
4.5	The Non-existence of Limit Cycles in Region III	107
Chapter Five	Conclusion	122
5.1	Graphical Illustrations	122
5.2	Physical Interpretation	133
5.3	Further Problems	135
Appendix A		137
Appendix B		142
Appendix C		147
References		150