METHODS FOR THE EVALUATION OF N-DIMENSIONAL INTEGRALS

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ABSTRACT

The method of undetermined coefficients has been used to construct integration formulae having special properties. Formulae have been developed which are third degree for general functions, but fifth or seventh degree for symmetric functions when the region of integration is the square or the circle. Further, we have established a third-degree formula for the cube and we make the generalization to the n-dimensional space, for $n \leq 6$.

Two criteria are given for optimizing formulae of a given type depending on a parameter - the minimax and least squares methods. In one dimension this approach gives formulae which are more reliable than Gaussian formulae for a general function, in the sense that they give good results for differentiable functions and seem to give better results for singular test integrals. The least squares method has been used to obtain analogous results for 2-dimensional integrals.

Series expansion has been used to evaluate some n-dimensional integrals and this can be a very powerful technique. The method adopted has been expansion of the integrand followed by term-by-term integration. The resulting series is then summed by Levin's method. We demonstrate the method by evaluating certain types of

multiple integrals (Lattice Green functions) to a high degree of accuracy.

Finally, we have used the method of undetermined coefficients to obtain formulae when the weight function is different from 1; in particular $x^{\frac{1}{2}}$.